

STANDARD
& POOR'S

Index Calculation Primer

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What is an index?

- An index is a single descriptive statistic that summarizes the relative change in an underlying group of variables.
- In an equity index, such as the S&P 500, the underlying variables are stocks.
- The main differences among indexes is the types of securities held and the weighting scheme.

Index Groupings

- There are many types of indexes, each trying to measure different groups of stocks:

Broad based

Small Cap
Mid Cap
Large Cap
Value
Growth
Geographic region

Narrow

Economic Sector
Industry

- Or any combination of the above.

Index Groupings

- These groupings are usually based on simple financial ratios.
- Size (small, mid, or large) is based on market cap, which is price times shares outstanding.
- Style (value or growth) is often based on book to price ratio, which is the company's common equity divided by its share price.

Index Weightings

- Index constituents can be either equal weighted, price weighted, or cap weighted.
- Lets say we wanted to form a new index comprised of the five largest cap stocks in the S&P 500 as of May 31, 2000.

	Sales	Shares	Price
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Equal Weighted

- Equal weighting would consist of giving each stock equal representation in the index. In this example that's a weight of 20%.
- To design such an index, we would designate some amount of fictional money (say \$10,000) to be invested in each stock. Then divide that amount by the stock price to get how many shares to buy. Lets call this number Index Shares.

	Price	Index Shares	Dollar Holdings
• CISCO SYSTEMS INC	56.938	175.629	10,000
• EXXON MOBIL CORP	83.312	120.031	10,000
• GENERAL ELECTRIC CO	52.688	189.797	10,000
• INTEL CORP	124.688	80.200	10,000
• MICROSOFT CORP	62.562	159.841	10,000

Price Weighted

- Price weighting would consist of buying an equal number of shares of each stock in the index.
- The higher the price, the more weight the stock has in the index.
- For example, Intel has twice the weight of Microsoft, even though Microsoft's market cap is larger than Intel's.
- The Dow Jones Industrial Average is price weighted because in 1890 (before computers) the easiest thing to do was to add up 12 prices and divide by 12.

	Price	Index Shares	Dollar Holdings
• CISCO SYSTEMS INC	56.938	10,000	569,380
• EXXON MOBIL CORP	83.312	10,000	833,120
• GENERAL ELECTRIC CO	52.688	10,000	526,880
• INTEL CORP	124.688	10,000	1,246,880
• MICROSOFT CORP	62.562	10,000	625,6200

Cap Weighted

- Cap weighting is weighting by market capitalization, which is shares times price.
- In this case index shares (how much one needs to hold to match an index) are the same as shares outstanding (the number of shares a company has issued).
- The S&P 500 Index is Cap weighted.

	Index Shares	Price	Market Cap
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Index Shares

- It was mentioned in the previous slide that Index Shares are the same as the company's shares outstanding. That is the case when a company first goes into an S&P index, but after that it may vary by up to 5%
- If a company changes its shares outstanding and the new number varies by more than 5% from the Index Shares, S&P will change the Index Shares immediately to reflect the change.
- If a company changes its shares outstanding and the new number varies by less than 5% from the Index Shares, S&P will not change the Index Shares until its next quarterly rebalancing.
- Whenever S&P changes the Index Shares of a company, hundreds of money managers have to effect equivalent changes, incurring transaction costs. Therefore S&P tries to limit the frequency of rebalancings.

Comparison of Returns

	Price t=0	Price t=1	Price t=2	Price t=3	Return
• CISCO SYSTEMS INC	56.938	60	61	62	8.89%
• EXXON MOBIL CORP	83.312	82	81	80	-3.98%
• GENERAL ELECTRIC CO	52.688	55	60	65	23.37%
• INTEL CORP	124.688	120	110	100	-19.80%
• MICROSOFT CORP	62.562	60	70	60	-4.10%
					0.88%
Equal Weighted Index	50000.00	50033.62	51834.61	50438.77	0.88%
Price Weighted Index	3801880.00	3770000.00	3820000.00	3670000.00	-3.47%
Cap Weighted Index	1954841.94	1965429.52	2037291.68	2004313.00	2.53%

- We see that how you weight the index makes a big difference in the returns.
- Price weighting gives most of the weight to Intel, so the index value goes down.
- Cap weighting gives most of the weight to GE, so the index value goes up.
- Equal weighting gives the same result as the average of the individual stock's returns.

The Divisor

- The index values are hard to compare because they all start at different numbers, not to mention being rather large.
- We can re-base these indexes by introducing what is called a divisor.
- This allows us to start the index at any value we like, lets say 100.
- The initial divisor is the time zero price of the index divided by the base level of the index (Cap example: $1,954,841.94 / 100 = 19,548.42$.)

	Price t=0	Price t=1	Price t=2	Price t=3	Returns
• Equal Weighted Index	50000.00	50033.62	51834.61	50438.77	0.88%
• Price Weighted Index	3801880.00	3770000.00	3820000.00	3670000.00	-3.47%
• Cap Weighted Index	1954841.94	1965429.52	2037291.68	2004313.00	2.53%
•					
					Divisor
• Equal Weighted Index	100.00	100.07	103.67	100.88	500.00
• Price Weighted Index	100.00	99.16	100.48	96.53	38,018.80
• Cap Weighted Index	100.00	100.54	104.22	102.53	19,548.42

Index Value Calculation

- The formal formula to calculate a cap weighted index value, such as the S&P 500 index value, is:

$$\text{Index Value} = 1/\text{divisor} * \text{SUM} (\text{Price}(i) * \text{Index Shares}(i))$$

where i goes from 1 to 500--representing each stock in the S&P 500.

- Market Value of the index is: $\text{SUM} (\text{Price}(i) * \text{Index Shares}(i))$
- So we have this fundamental relationship:
- **Index Value = Market Value / Divisor**

Handling Share Changes

- Let's say that at time $t=2$ Exxon Mobil doubles its number of shares outstanding to pay for a big acquisition.
- This has no effect on the equal and price weighted indexes because the index shares used for these indexes do not change, but the index shares for the cap weighted index is the number of shares the company has outstanding, so the cap weighted index shares also doubles.
- We will have to adjust the divisor to negate the effect of this.

	Equal Sh	Price Sh	Cap Sh $t<2$	Cap Sh $t\geq 2$
CISCO SYSTEMS INC	175.630	10000	7000.94	7000.94
EXXON MOBIL CORP	120.031	10000	3481.02	6962.04
GENERAL ELECTRIC CO	189.797	10000	9882.34	9882.34
INTEL CORP	80.200	10000	3348.99	3348.99
MICROSOFT CORP	159.841	10000	5242.04	5242.04

New Divisor

- Dealing only with the cap weighted index, before the change in shares we had the following:

	Price t=0	Price t=1	Price t=2	Price t=3
Cap Weighted Index	1954841.94	1965429.52	2037291.68	2004313.00
Cap Weighted Index	100.00	100.54	104.22	102.53

- After the change in shares, but before we adjust the divisor, we have:

	Price t=0	Price t=1	Price t=2	Price t=3
Cap Weighted Index	1954841.94	1965429.52	2319254.37	2282794.67
Cap Weighted Index	100.00	100.54	118.64	116.78

- After we change the divisor, we get more reasonable numbers (why did the return still go down?):

	Price t=0	Price t=1	Price t=2	Price t=3
Cap Weighted Index	1954841.94	1965429.52	2319254.37	2282794.67
Cap Weighted Index	100.00	100.54	103.60	101.97

- How do we calculate the new divisor?

Answers

- Why did the return still go down? Since we changed the divisor, shouldn't the Index Value be the same as if the Index Shares had not changed?
- No, the divisor makes the Index Value based on the old Index Shares and old divisor equal to the Index Value based on the new Index Shares and new divisor (In our example, at time=1).
- At time=2, Exxon Mobil now has twice as much weight in the index as at time=1, so the fact that its hypothetical return was negative will drag the index down.
- The shares of Exxon Mobil changes at time=2, so at time=1, define MV(old) as the market value of the index using the Index Shares numbers for time < 2 and MV(new) as the market value of the index using the Index Shares numbers for time >= 2. We want the Index Value to be the same under both scenarios. We know the old divisor, so we can solve for the new divisor.
- $MV(\text{old})/\text{Divisor}(\text{old}) = \text{Index Value}(\text{old}) = MV(\text{new})/\text{Divisor}(\text{new})$, which yields:
- **$\text{Divisor}(\text{new}) = MV(\text{new}) / \text{Index Value}(\text{old})$**

Stock Splits

- How does the index handle stock splits?
- First, what is a stock split?
- Research has been done showing that many people like to buy stocks in the \$20 to \$60 or so price range, as opposed to say \$200 stocks.
- A company with a stock price of \$200 may want to issue a 4-for-1 stock split.
- This will result in a four-fold increase in the number of shares outstanding, but since the market value of the company has not changed, financial theory has it that the stock price should fall proportionately (by three-fourths to \$50).
- S&P adjusts the company's Index Shares and price for all splits.
- **The divisor would not need to change, however, since the split had no effect on the market cap of the company.**

Dividends

- Though currently not a hot topic, stocks can issue three kinds of dividends: Cash dividends, stock dividends, and special dividends.
- Cash and stock dividends both are funded from retained earnings. This means that these dividends have no effect on the market capitalization of the company, thus the divisor does not change.
- Special dividends require a change in the divisor because the money does not come out of retained earnings and thus does change the market capitalization of the company.

Descriptive Calculations

- Now that we know how to set up and maintain the index, let's do something more fun, like calculate some descriptive statistics.
- A good place to start is with the price-to-earnings ratio.
- Before we saw that $\text{Market Value} = \text{SUM} (\text{Price}(i) * \text{Index Shares}(i))$
- Likewise, $\text{Earnings} = \text{SUM} (\text{EPS}(i) * \text{Index Shares}(i))$
- Earlier we saw that $\text{Index Value} = \text{Market Value} / \text{Divisor}$, where Market Value is the sum of the entire market value of all the stocks in the index and Index Value can be interpreted as the “per share” equivalent. In other words, just as the divisor allows us to re-base the index at any level we want, it also allows us to say what “one share” of the index is worth.
- Thus, we use the divisor to find EPS as follows:
- $\text{EPS}(\text{index}) = 1/\text{Divisor} * \text{SUM}(\text{Earnings}(i))$, or equivalently
- **$\text{EPS}(\text{index}) = 1/\text{Divisor} * \text{SUM}(\text{EPS}(i) * \text{Index Shares}(i))$**

How to Calculate P/E?

	Price	Index Shares	EPS	P/E RATIO	MKVAL	EARNINGS
CISCO SYSTEMS INC	56.938	7000.939	0.53	107.43	398619.438	3710.49722
EXXON MOBIL CORP	83.312	3481.021	3.73	22.04	290010.81	12984.20635
GENERAL ELECTRIC CO	52.688	9882.338	1.25	42.15	520680.63	12352.92251
INTEL CORP	124.688	3348.987	3.07	40.61	417578.44	10281.38776
MICROSOFT CORP	62.562	5242.042	1.70	36.80	327952.625	8911.470221

- Given the above data, what is the P/E Ratio for the index?

A) 49.85 = $1/5 * (107.43 + 22.04 + 42.15 + 40.61 + 36.80)$

B) 51.29 = $(MKVAL(i) * PERATIO(i)) / SUM (MKVAL(i))$

C) 40.52 = $SUM (MKVAL(i)) / SUM (Earnings(i))$

Don't Average Ratios

- Answer A is a straight equal weight of the 5 individual ratios, but we are leery of this answer because we know the index is cap weighted. Besides the Cisco P/E of 107 seems to throw the whole number off.
- Answer B seems to get us closer because this is clearly a market weight calculation, but the Cisco P/E still seems to skew up the average since it is over 20% of the index.
- Answer C is very appealing because by summing the numerator and denominator separately, then averaging we completely avoid the problem of outliers. (This method also deftly handles negative P/E ratios.)
- In this example, “C” stands for Correct!
- **Bonus Questions: Is this P/E equal weighted or market weighted?**

Implicit Market Weighting

- Upon first glance Answer C does not appear to be market weighted because the weights do not show up in the calculation (as they do in Answer B).
- Maybe it will be easier to see if we instead write the P/E formula as:
$$\frac{\text{MKVAL}(\text{index})}{\text{Earnings}(\text{index})}$$
 instead of
$$\frac{\text{SUM}(\text{MKVAL}(i))}{\text{SUM}(\text{Earnings}(i))}$$
, though they are equal!
- This is equivalent to the formula in Answer C because we simply multiply each constituent in the denominator and numerator by its associated Index Shares figure.
- This highlights the fact that to arrive at earnings, we need to know Index Shares; and these Index Shares are different for each company. Indeed, it's the Index Shares that market weight the calculation.
- It just turns out that the easiest method (using each company's total earnings) is the correct one.
- If we wanted to equally weight the P/E ratio we would need to use the "equal shares" we calculated at the beginning of the presentation and find the new "equal weighted earnings total". (This would actually be much harder to do.)

Other Calculations

- Sales Per Share is another example:

$$\text{Sales Per Share(index)} = 1/\text{divisor} * \text{SUM(Sales(i))}$$

- In fact, almost any data item can be put into this generic formula:

$$\text{X Per Share(index)} = 1/\text{divisor} * \text{SUM(X(i))}$$

where X is a non-per share item.

- If X is a per share item:

$$\text{X Per Share(index)} = 1/\text{divisor} * \text{SUM(X(i) * Index Shares(i))}$$

Contribution Analysis

- We saw earlier that on a cap weighted basis the index returned 2.53% from time=0 to time=3. What is the contribution of each stock?
- If we use the beginning time period weights for each stock multiplied by its return, we get the last column, which shows the weighted return of each stock.
- The sum of the last column gets us back to 2.53%
- So we see that GE was the largest contributor of return by adding 6.22% and Intel was the largest drag on index performance, subtracting 4.23%.

	Return	MKVAL	Weight	Contribution
CISCO SYSTEMS INC	8.89%	398619.438	0.2039139	1.81%
EXXON MOBIL CORP	-3.98%	290010.813	0.1483551	-0.59%
GENERAL ELECTRIC CO	23.37%	520680.625	0.2663543	6.22%
INTEL CORP	-19.80%	417578.438	0.2136124	-4.23%
MICROSOFT CORP	-4.10%	327952.625	0.1677643	-0.69%
				2.53%

Conclusion

- This presentation has covered just about all the math one would need to know to start and maintain an index.
- We also showed the effects of different weighting schemes on index returns.
- Finally, we showed how to calculate descriptive statistics for an index as well as analyze which stocks provided the most returns to the index.

PART 2

The following slides are from a separate presentation and have been included since they may provide the reader with more specific information on how Standard & Poor's calculates ratios.



Calculating Index Ratios

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Example of P/E Calculations

Various P/Es for a three stock portfolio

COMPANY	PRICE	EPS	SHARES	MKTCAP	EARNINGS	P/E
X	2.00	0.25	1000	2000	250	8.00
Y	10.00	0.50	1000	10000	500	20.00
Z	7.00	-0.05	1500	10500	-75	-140.00
<i>Totals</i>				<u>22500</u>	<u>675</u>	<u>-112.00</u>
AVERAGE OF P/E RATIOS						-37.33
WEIGHTED AVERAGE OF P/E RATIOS						-55.73
AVERAGE OF P/E RATIOS (EXCLUDING NEGATIVES)						14.00
ATTRIBUTABLE EARNINGS P/E						33.33

The P/E for company Z is an outlier and makes most of the above P/E averages suspect.

Methodology Matters

- The preceding extreme example shows that different methods of calculating a portfolio or index P/E results in very different numbers.
- What criteria should we use to decide which is the best methodology?

Attributable Earnings

- Let's define **Attributable Earnings** as the EPS of a company multiplied by the number of shares one holds of that company.
- The sum of this value for each stock would be the earnings attributable to the portfolio.
- The market value of the portfolio divided by the earnings attributable to the portfolio is the best way of calculating a portfolio (or index) P/E ratio.

Effect of Outliers

- A company with 10,000 shares outstanding that sells for \$10 a share and has EPS of \$0.01 has a P/E of 1000.
- Even if the company has a small market cap, an outlier this large could still have a warping effect.
- In the next slide, we will show two examples of how far the P/E can get distorted by outliers.

Two Examples of Outliers

Two companies
of equal size:

Shares	EPS	Price	P/E	Market Cap	Attributable Earnings
1000	0.01	10	1000.00	10000	10
500	0.75	20	26.67	10000	375
Equal and Weighted Average P/E					513.33
Attributable Earnings P/E					51.95

Shares	EPS	Price	P/E	Market Cap	Attributable Earnings
1000	0.01	10	1000.00	10000	10
2500	0.75	20	26.67	50000	1875
Equal Average P/E					513.33
Weighted Average P/E					188.89
Attributable Earnings P/E					31.83

Two companies
of different size:

How S&P Calculates Index P/Es

We calculate the earnings attributable to the S&P 500 Index as follows:

- $\text{Index P/E} = \frac{\sum \text{Market Cap}(i)}{\sum \text{Earnings}(i)}$
 - $\text{Market Cap}(i) = \text{Price}(i) * \text{Index Shares}(i)$
 - $\text{Earnings}(i) = \text{EPS}(i) * \text{Index Shares}(i)$

Since the S&P 500 is market cap weighted, Index Shares(i) is the number of shares company i has outstanding.

Calculating P/Es of Portfolios

To calculate a portfolio P/E that is most comparable to the benchmark P/E, use the same formula:

- Portfolio P/E = $\sum \text{Market Cap}(i) / \sum \text{Earnings}(i)$
 - Market Cap(i) = Price(i) * Portfolio Shares(i)
 - Earnings(i) = EPS(i) * Portfolio Shares(i)

Other Price-Based Ratios

- This method works for other ratios as well.
- Portfolio P/B = $\sum \text{Market Cap}(i) / \sum \text{Book Value}(i)$
 - $\text{Market Cap}(i) = \text{Price}(i) * \text{Portfolio Shares}(i)$
 - $\text{Book Value}(i) = \text{Book Value/Sh}(i) * \text{Portfolio Shares}(i)$
- Any ratio can be calculated this way, even non price-based ratios. In the next slide we will see how to calculate the Current Ratio of a portfolio.

Portfolio Current Ratio

- Current Ratio = $\sum \text{Current Assets}(i) / \sum \text{Current Liabilities}(i)$
 - Current Assets(i) = CA/Sh(i) * Portfolio Shares(i)
 - Current Liabilities(i) = CL/Sh(i) * Portfolio Shares(i)
- We divide CA by the number of shares the company has outstanding and then multiply by the number of shares held by the portfolio to get the CA attributable to the portfolio.

Attributable Growth Rate

This example shows three methods of calculating an earnings growth rate:

Company	Shares	Price	Value of Holdings	EPS(t)	Earnings(t)	Exp Growth	EPS(t+1)	Earnings(t+1)	
A	500	100	\$ 50,000.00	1.00	500	30%	1.30	650	
B	200	50	\$ 10,000.00	1.50	300	20%	1.80	360	
C	500	20	\$ 10,000.00	2.00	1000	10%	2.20	1100	
					<u>1800</u>			<u>2110</u>	
Attributable Growth:	$(2110/1800) - 1 =$					17.22%			
Weighted Average:	$(5/7)*0.3 + (1/7)*0.2 + (1/7)*0.1 =$					25.71%			
Share Wt Average:	$(5/12)*0.3 + (2/12)*0.2 + (5/12)*0.1 =$					21.05%			

- The attributable growth rate is the only measure that accurately portrays the earnings growth rate that a portfolio manager (or index) could claim.
- This methodology could be used for Dividends Per Share, Cash Flow Per Share, or anything else.

The Moral

- The moral of this story is that one should not take an average of ratios, but rather take a ratio of sums.
- Equally important, one should consider the amount of the data item in question attributable to the portfolio. In other words, weight it properly based on number of shares held.

“Attributable” Advantages

- Its the same methodology that most index providers use.
- Methodology is more mathematically sound than averaging ratios.
- Allows better treatment of negative values and outliers than simply ignoring them.